

## Appendix

$$f(r, z) = \sum_{m=0}^3 \sum_{n=0}^3 a_{m,n} r^m z^n, \quad (1)$$

$$\begin{aligned} f_{k,l} &= [f(r, z)]_{k,l} = \phi_{k,l}, \\ \left( \frac{\partial f}{\partial r} \right)_{k,l} &= \left( \frac{\partial \phi}{\partial r} \right)_{k,l}, \\ \left( \frac{\partial f}{\partial z} \right)_{k,l} &= \left( \frac{\partial \phi}{\partial z} \right)_{k,l}, \\ \left( \frac{\partial^2 f}{\partial r \partial z} \right)_{k,l} &= \left( \frac{\partial^2 \phi}{\partial r \partial z} \right)_{k,l}, \end{aligned} \quad (2)$$

$$k = i, i+1, \quad l = j, j+1, \quad (3)$$

$$T_{i,j}^T = \begin{pmatrix} z_{,\eta} & -z_{,\xi} \\ -r_{,\eta} & r_{,\xi} \end{pmatrix}_{i,j}, \quad J_{i,j} = (r_{,\xi} z_{,\eta} - r_{,\eta} z_{,\xi})_{i,j}, \quad (4)$$

$$\begin{pmatrix} \frac{\partial \phi}{\partial r} & \frac{\partial \phi}{\partial z} \end{pmatrix}_{i,j} = J_{i,j}^{-1} T_{i,j}^T \begin{pmatrix} \frac{\partial \phi}{\partial \xi} \\ \frac{\partial \phi}{\partial \eta} \end{pmatrix}_{i,j}, \quad (5)$$

$$\begin{aligned} \left( \frac{\partial \phi}{\partial \xi} \right)_{i,j} &= (\phi_{i+1,j} - \phi_{i-1,j})/2, \\ \left( \frac{\partial \phi}{\partial \eta} \right)_{i,j} &= (\phi_{i,j+1} - \phi_{i,j-1})/2, \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial^2 \phi}{\partial r \partial z} &= (\phi_{,r})_{,\xi} \xi_{,z} + (\phi_{,r})_{,\eta} \eta_{,z} \\ &= \phi_{,\xi\xi} \xi_{,r} \xi_{,z} + \phi_{,\xi\eta} (\xi_{,z} \eta_{,r} + \xi_{,r} \eta_{,z}) + \phi_{,\eta\eta} \eta_{,r} \eta_{,z}, \end{aligned} \quad (7)$$

$$\begin{aligned} \phi_{,\xi\xi} &= \phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}, \\ \phi_{,\eta\eta} &= \phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}, \\ \phi_{,\xi\eta} &= (\phi_{i+1,j+1} - \phi_{i-1,j+1} - \phi_{i+1,j-1} + \phi_{i-1,j-1})/4, \end{aligned} \quad (8)$$

$$f(y) = 0, \quad \nabla f(y) \times (x^0 - y) = 0, \quad (9)$$

$$\begin{aligned} \delta_1 &= -f(x^k) \frac{\nabla f(x^k)}{\nabla f(x^k) \cdot \nabla f(x^k)}, \\ x^{k+1/2} &= x^k + \delta_1, \\ \delta_2 &= (x^0 - x^k) - \frac{(x^0 - x^k) \cdot \nabla f(x^k)}{\nabla f(x^k) \cdot \nabla f(x^k)} \nabla f(x^k), \\ x^{k+1} &= x^{k+1/2} + \delta_2, \end{aligned} \quad (10)$$

where  $k = 0, 1, 2, \dots$ .

$$|\delta_1|^2 + |\delta_2|^2 < 10^{-6} \Delta r \Delta z, \quad (11)$$

$$f(r, z) = \sum_{m=0}^1 \sum_{n=0}^1 a_{m,n} r^m z^n, \quad (12)$$

$$|\nabla \phi| = 1, \quad (13)$$

$$\max (D_{ij}^{-x} \phi, -D_{ij}^{+x} \phi, 0)^2 + \max (D_{ij}^{-y} \phi, -D_{ij}^{+y} \phi, 0)^2 = 1, \quad (14)$$

$$\begin{aligned} D_{ij}^{-x} \phi &= \frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta x}, \\ D_{ij}^{+x} \phi &= \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x}, \end{aligned} \quad (15)$$

$$(a^T Q a) \phi^2 + (2a^T Q b) \phi + (b^T Q b) = 1, \quad (16)$$

$$\begin{aligned} Q &= (P P^T)^{-1}, \\ P &= \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}, \\ P_1 &= (x - x_A) / |x - x_A|, \\ P_2 &= (x - x_B) / |x - x_B|, \\ a &= (a_1 \ a_2) = \begin{pmatrix} 1 & 1 \\ s_1 & s_2 \end{pmatrix}, \\ b &= (b_1 \ b_2) = \begin{pmatrix} -\phi_A & \phi_B \\ s_1 & s_2 \end{pmatrix}. \end{aligned} \quad (17)$$

$$\begin{aligned} (a_1 \phi + b_1) &\geq (P_1 \cdot P_2^T)(a_2 \phi + b_2), \\ (a_2 \phi + b_2) &\geq (P_1 \cdot P_2^T)(a_1 \phi + b_1), \end{aligned} \quad (18)$$

$$P \nabla \phi = \nu, \quad \nu = (\nu_1 \ \nu_2), \quad (19)$$

$$\begin{aligned} (P^{-1} \nu) \cdot (P^{-1} \nu) &= 1, \\ \text{or } (P^{-1} \nu)^T \cdot (P^{-1} \nu) &= 1, \\ \text{or } \nu^T (P P^T)^{-1} \nu &= 1, \end{aligned} \quad (20)$$

$$\nu_1 = (\phi - \phi_A) / s_1, \quad \nu_2 = (\phi - \phi_B) / s_2, \quad (21)$$

$$\Delta t < \min_{i,j} \left[ \frac{\Delta r}{u}, \frac{\Delta z}{v}, \sqrt{We \frac{\rho_1 + \rho_2}{8\pi}} h^{3/2}, \frac{Re}{2} \frac{\rho^n}{\mu^n} \left( \frac{1}{\Delta r^2} + \frac{1}{\Delta z^2} \right)^{-1}, \sqrt{\frac{2h}{|F|}} \right], \quad (22)$$

$$\Delta t_0 < \min_{i,j} \left[ \sqrt{We \frac{\rho_1 + \rho_2}{8\pi}} h^{3/2}, \frac{Re}{2} \frac{\rho^n}{\mu^n} \left( \frac{1}{\Delta r^2} + \frac{1}{\Delta z^2} \right)^{-1} \right], \quad (23)$$

$$\Delta t' < \min_{i,j} \left[ \frac{\Delta r}{u}, \frac{\Delta z}{v}, \sqrt{\frac{2h}{|F|}} \right], \quad (24)$$

$$\Delta t = \min(\Delta t_0, \Delta t'), \quad (25)$$

$$\kappa(\phi) = J^{-1} \nabla_{\Xi} \cdot \left( gT \frac{T^T \nabla_{\Xi} \phi}{|T^T \nabla_{\Xi} \phi|} \right), \quad (26)$$

$$\nabla_{\Xi} \phi = \left( \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2}, \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2} \right), \quad (27)$$

$$\kappa_{critical} = 1.d0/\epsilon, \quad (28)$$

$$\kappa(\phi)_{i,j}, \kappa(\phi)_{i+1,j}, \kappa(\phi)_{i,j+1}, \text{ or } \kappa(\phi)_{i+1,j+1} > \kappa_{critical}, \quad (29)$$

$$\rho_1 = 1000 \text{ Kg/m}^3, \quad \mu_1 = 10. \times 10^{-3} \text{ Kg/m} \cdot \text{sec}, \quad \sigma = 0.032 \text{ Kg/sec}^2, \quad (30)$$

$$Re = 4.8, \quad We = 9.6 \quad (31)$$

$$\rho_1 = 1070 \text{ Kg/m}^3, \quad \mu_1 = 3.5 \times 10^{-3} \text{ Kg/m} \cdot \text{sec}, \quad \sigma = 0.032 \text{ Kg/sec}^2. \quad (32)$$